Proper time and Minkowski structure on causal graphs

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Abstract

For causal graphs we propose a definition of proper time which for small scales is based on the concept of volume, while for large scales the usual definition of length is appied. The scale where the change from "volume" to "length" occurs is related to the size of a dynamical clock and defines a natural cut-off for this type of clock. By changing the cut-off volume we may probe the geometry of the causal graph on different scales and thereby define a continuum limit. This provides an alternative to the standard coarse graining procedures. For regular causal lattices (like e.g. the 2-dim. light-cone lattice) this concept can be proven to lead to a Minkowski structure. An illustrative example of this approach is provided by the breather solutions of the Sine-Gordon model on a 2-dimensional light-cone lattice.

University of Freiburg
February 2001
preprint THEP 01/03

1 Introduction

Recently, there has been an increased interest in discrete causal structures as models for spacetime at small scales [1, 2, 3, 4, 5, 6]. In some cases, the dynamics which generates these spacetime structures assigns to events a time-labeling which is unphysical in the sense that it fixes equal time relations and rather resembles a Newtonian spacetime than a Minkowskian one. This time-labeling, however, is not expected to be related to the physical time which an intrinsic observer would experience and which should exhibit a Minkowski or Lorentz structure. (In [7] an axiom of "discrete general covariance" requires that physical quantities do not depend on this labeling.)

To recall an expression of Einstein, the physical time is "the time you can read from a clock". A clock is a physical system which takes part in the dynamics and which allows to identify and count a characteristic time scale. In the ideal case the evolution of this system is periodic and the number of "ticks" between two events on the world-line of this clock is equal to the length (the proper time) of this section of the world-line. Furthermore, a clock should have a negligible influence on the spacetime structure and should be small compared to the scale of geometrical variations of the underlying geometry. In order that different types of clocks define the same geometry (up to a change of scale), the fundamental dynamics should be universal. In continuum physics this property is guaranteed by local Lorentz invariance of the dynamics as expressed, e.g., in the wave operator and the Dirac operator for bosonic and fermionic fields, respectively.

Unfortunately, discretized analogues of the wave operator or the Dirac operator are only known for special cases: In [8] the Sine-Gordon model is studied on a light-cone lattice (see also sect. 5) and Feynman gives a prescription to obtain the 2-dimensional Dirac propagator from a random walk prescription [9] which is related to the 6-vertex model in statistical mechanics [10]. For unoriented graphs, which serve as discretizations of euclidean spacetime, the situation is different in that an analogue of the Laplacian is well known and extensively studied in the literature (for a collection of results on the spectrum of the graph laplacian see e.g. [11].) For the same reason, the "summation over paths"-representation of propagators or Green functions in Minkowski space is by far not as well understood as in euclidean spaces (see e.g. [12]). It is known that a summation over timelike paths will not lead to the correct expressions, non-physical paths violating causality have to be taken into account. While the physical paths formally are "weighted" by a phase, the unphysical contributions are damped by real factors. However, this prescription is derived from the formal expression

$$G(x,y) \;=\; \sum_{\text{paths } x\to y} \exp\left(\mathrm{i}\int \mathrm{d}\tau \sqrt{-\dot{x}^2}\right) , \qquad (1)$$

and any attempts to give it a precise meaning which also works for discretized versions (e.g. on causal graphs) have so far failed. It should be emphasized, however, that the real dynamics is not given by some particle or field which propagates *on* an existing physical spacetime but the dynamics should be such that spacetime *and* matter are generated simultaneously (a possible mechanism for this is indicated in [7]).

If the microscopic dynamics of fields on a causal graph would be known, we might be able to derive a definition of proper time using the propagator function (see sect. 3, where we will also show why the definition of proper time as "the number of links" of a timelike path is inadequate for many cases). There is one property, however, which any definition of proper time should satisfy if it is to lead to a Lorentz structure: in flat Minkowski spacetime the geodesic distance between any two events, (a, b), should be in on-to-one correspondence to Alexandrov volumes $V[a, b]$ (i.e. the number of events z such that $\{a_i < z < b_i\}$, where "$<$" denotes the causal ordering of events). This defines an equivalence relation on the space of event pairs, and although the volume is not the measure of proper time, it can be used to define such a measure (sect. 2).

In general relativity we require that each event has a neighborhood such that within this neighborhood special relativity is approximated to a given accuracy (this excludes the singularities in the center of black holes etc.). Especially, the world volume spanned by the spacial extend and the unit of time of clocks should be within such a neighborhood. On the other hand, we expect the spacetime structure to become non-trivial again for very small scales and if we assume spacetime to be discrete at Planck scale it remains questionable, if clocks can be arbitrarily small. Any dynamical clock has an intrinsic cut-off corresponding to its characteristic time unit. If there exist clocks that can measure time down to Planck scale and resolve the discreteness of spacetime, then it is not obvious at all that these clocks will define the same Minkowski structure as a clock for which the smallest measurable time scale is within the region where the approximation of spacetime by a flat Minkowski space is reasonable.

We present a definition of proper time which associates to each clock a characteristic Alexandrov volume γ which sets the time unit for this clock: Two timelike events (a, b) are separated by a proper time of "one tick" of this clock, if the Alexandrov volume corresponding to these two points is γ. Two timelike events (a, b) are separated by "two ticks", if there exists a third event c such that the Alexandrov volumes of (a, c) and (c, b) both are equal to γ and if there is no other event c', such that $V[a, c'] > V[a, c]$ and $V[c', b] > V[c, b]$ (for details see sect. 4). Different types of clocks will be associated with different γ, but γ should not be too large, as in this case the clock will not resolve the large scale fluctuations of spacetime geometry, and γ should not be too small, as in this case the clock will resolve the discreteness of spacetime. Within these limits a change of γ is merely a change of time scale and we probe the continuum theory. Two such clocks will define the same geometry, and we refer to such clocks as "standard" clocks. In principal, γ can be arbitrary small (even $\gamma = 0$, in which case we recover the proper time definition related to the "number of links"), but in this case even the large scale structure defined by such a clock may be different from the one defined by a standard clock.

This paper is organized as follows. In sect. 2 we give some definitions related to causal graphs and causal sets and recall some well known relations between volume and proper time on flat Minkwoski spaces. In sect. 3 we show that the definition of proper time as the number of links of a timelike path does not lead to a Minkowski structure on special causal graphs as, e.g., the light-cone lattice. An example will illustrate how the knowledge of the "summation over path"-prescription might lead to a definition of proper time. In sect. 4 we present our definition of proper time which is based on Alexandrov sets and in sect. 5 we will use the breather solutions of the Sine-Gordon theory to illustrate the idea of dynamical clocks on causal graphs in a special case. Some remarks will conclude this paper.

2 Definitions and preliminary remarks

We start by giving some definitions related to causal graphs.

DEF.: A *simple directed graph* is a non-reflexive, asymmetric relation E on a set V, the set of vertices (or events).

Hence, if $(a,b) \in E$, then $a \neq b$ and $(b,a) \notin E$. We say that a and b are connected by a causal link (or edge). Instead of directed graphs one also speaks of oriented graphs. Furthermore, two events cannot be connected by more than one link. We will always assume V to be countable.

DEF.: A *directed path* on a simple directed graph from vertex a to vertex b is a succesion of N pairs $(c_i, c_{i+1}) \in E$ $(i = 0, ..., N-1)$ such that $c_0 = a$ and $c_N = b$. This path is said to be of *length* N. If $a = b$ the path is said to be *closed*.

DEF.: A *causal graph* is a simple directed graph such that there are no closed directed paths.

A causal graph defines a causality relation "$<$": We write $a < b$, iff there exists a directed path from vertex a to vertex b. This relation is asymmetric, non-reflexive and transitive. Given a causality relation, we can speak of timelike and spacelike events, the set of future events, and the set of past events, etc.

DEF.: For two vertices a and b we define the *Alexandrov set* to be $[a,b] = \{z | a < z < b\}$. The *volume* of the Alexandrov set (the number of its elements) will be denoted by $V[a,b]$.

In the following, we will require the property of local finiteness, i.e., for any pair of events the Alexandrov set is finite. Under these conditions a causal graph uniquely defines a causal set [3]. It should be noted, however, that causal sets and causal graphs are not equivalent. Although one can construct a unique causal graph from a causal set by "deleting" all relations which follow from transitivity, this causal graph might be different from the graph one has started with, as we do not require that the links in E are free of any transitivity relations. (There also seems to be a slight difference between causal sets and causal graphs related to the interpretation of the spacetime structure: For causal sets it is sometimes assumed that the number of events which are "lightlike" is of measure zero [2]; this is not the case for causal graphs.)

We now recall some well-known facts about the volume of Alexandrov sets and its relation to proper time in standard d-dimensional Minkowski space. We will use the convention that $(a-b)^2 > 0$ implies that a and b are timelike (and we will use units such that $c = 1$). $\tau(a,b) = \sqrt{(a-b)^2}$ will be the proper time distance between a and b.

If (a_1, b_1) and (a_2, b_2) are two pairs of timelike events in flat Minkowski space such that $\tau(a_1, b_1) = \tau(a_2, b_2)$, then the volumes of the corresponding Alexandrov sets are equal, $V[a_1, b_2] = V[a_2, b_2]$, and vice versa: equal volume of the Alexandrov sets implies equal proper time distance. This relation expresses the fact that the Alexandrov volume as well as the proper time are Lorentz invariants.

Let c and d be two spacelike events (i.e. $(c-d)^2 < 0$), then

$$(c-d)^2 \; = \; -\,\tau(a,b)^2 \, ,$$

where a and b are two timelike events such that c and d are in $[a,b]$ and such that there is no other Alexandrov set with this property having a smaller volume. (In more than two dimensions, a and b are not uniquely determined by c and d but different choices for (a,b) will have the same proper time distance.) Hence, once we have determined the timelike distances also the spacelike distances and the geometry are fixed. Therefore, in the following we will only be concerned with the definition of proper time and not with the determination of the full metrical structure. Note, however, that the relations above do not hold in curved spacetime.

Obviously, the volume of Alexandrov sets is not the *measure* of proper time (apart from "spacetime" dimension 1), as the relation between volume and time is non-linear and dimension dependent. However, we can reconstruct the proper time (up to an overall factor) from the knowledge of the Alexandrov volumes. This procedure is independent of the dimension and well known so that we only sketch the idea. Once we know how to double or bisect a certain distance the rest follows from suitable iteration of these steps.

For two timelike events a and b consider all events c such that $V[a,c] = V[c,b]$.

Among this set choose c such that $V[a,c]$ is maximal. Then c lies on the geodesic line connecting a and b and $\tau(a,c) = \tau(a,b)/2$. Similarly, consider all events d such that $V[a,b] = V[b,d]$ and choose d in this set such that $V[b,d]$ ist maximal, then $\tau(a,d) = 2\tau(a,b)$.

The *proper time distance* between two timelike events is equal to the maximal proper time length of a timelike path connecting these two events.

3 Mathematical and "propagator" proper time on causal graphs

In this section, we will investigate two suggestive definitions of proper time for causal graphs; one is the so-called mathematical proper time related to the number of links of a directed path and the other follows from a propagation prescription for a point particle on a causal graph. Applied to the simple example of a 2-dimensional light-cone lattice, both definitions fail to reproduce the usual Minkowski structure.

Def.: The *mathematical proper time* of a directed path from event a to event b is equal to the number of links N, i.e., the length of this path. The *mathematical proper time distance* between two timelike events a and b is equal to the maximal length of a directed path from a to b.

Let us now apply this definition to the 2-dimensional light-cone lattice (fig.1). All events on a fixed t-slice within the future light-cone have the same mathematical proper time distance from the event a. One might object that the graphical embedding of the light-cone lattice in the 2-dimensional plane of the paper is irrelevant for the intrinsic properties and misleading and that a suitable coordinate transformation will even make the equal time slices of standard Minkowski spacetime look horizontal. However, as we shall see in sect. 5, the dynamical proper time as experienced by an intrinsic observer and his clocks will show the familiar hyperbolic curves and has nothing to do with the horizontal equal time slices of the light-cone lattice.

Next, we show how to derive a definition of proper time from a random walk process of a point-like particle on the causal graph. This process does not represent the dynamics of real physical particles, as in this case the paths are "weighted" by phases instead of simple probabilities and non-causal paths have to be taken into account. The concept merely serves as an example how the dynamics of a particle may be used to define a proper time.

For two timelike events a and b let

$$G(a,b) \;=\; \sum_{\text{paths } a\to b} \mathrm{e}^{-\mu L}\,, \tag{2}$$

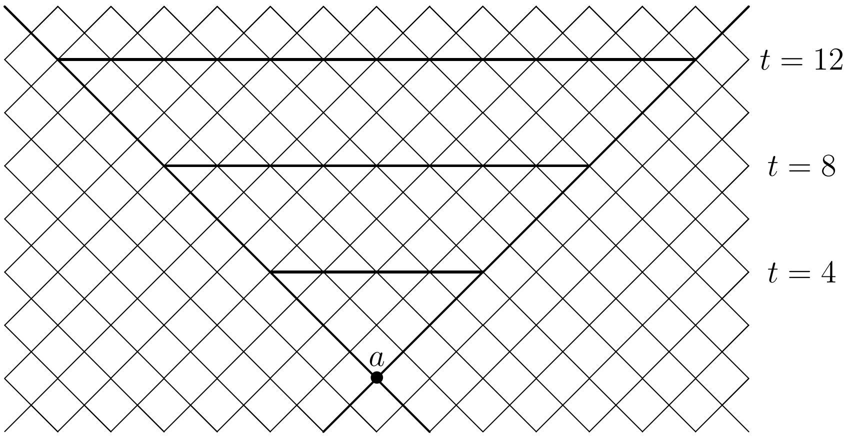

Figure 1: Equal time slices for the mathematical distance on the light-cone lattice

where L is the length of the oriented path from a to b. The summation extends over all directed paths from a to b and each path is weighted by a factor depending on its length only. (This is reminiscent of the definition of the euclidean Green function on undirected graphs.) We define:

Two pairs of events (a_1, b_1) and (a_2, b_2) have the same *propagator proper time distance*, iff $G(a_1, b_1) = G(a_2, b_2)$.

In complete analogy to the prescription sketched in the previous section for Alexandrov sets, we now can derive a proper time distance (up to an overall factor) from these "Green functions" $G(a, b)$. However, for the following argument we will only need the equal time slices of this definition, i.e., for a given a the set of events b for which $G(a, b)$ is constant.

Let us consider again the light-cone lattice and let a correspond to the point $(0, 0)$. All paths from $(0, 0)$ to a future event (x, t) have the same length t. Furthermore, the number of paths from $(0, 0)$ to (x, t) is simply given by a binomial coefficient. We are interested in the asymptotic behavior and make the usual gaussian approximation to obtain

$$G((0,0),(x,t)) \;\simeq\; \frac{2^t}{t} \exp\left(-2\frac{x^2}{t} - \mu t\right) . \tag{3}$$

An equal time slice is given by the set of events for which G is constant. The result depends on the value of μ: For $\mu < \ln 2$, the function G increases with increasing t and we "almost" obtain a hyperbolic structure, but with logarithmic corrections and a t-dependent center. For $\mu > \ln 2$, vertices of equal time are distributed along the section of a "distorted" circle which rather resembles the euclidean case.

4 Definition of proper time from Alexandrov sets

We now describe how to obtain the proper time of a directed path using Alexandrov sets.

We choose some reference volume γ and define $\tau_\gamma(a,b) = 1$, iff the Alexandrov volume $V[a,b] = \gamma$. Hence, γ defines the unit of proper time. For the moment, we will keep γ fixed and postpone the discussion about different and suitable choices of γ to the second part of this section. In terms of physics, we associate γ with a cut-off for a dynamical clock. γ represents the world-volume of this clock for one "tick". Such a "γ-clock" will not be able to resolve time on a scale smaller than $\tau_\gamma = 1$.

Now, let C be a directed path from x to y and $V[x,y] \gg \gamma$. The proper time of C between the events x and y is determined as follows: Choose events $z_0, z_1, ..., z_N$ on C such that (i) $z_0 = x$, (ii) $V[z_N, y] < \gamma$ and (iii) $V[z_i, z_{i+1}] = \gamma$. If such events can be found we say that the proper time (in units defined by γ) of the section of C between x and y is between N and $N+1$. A better resolution cannot be achieved with this type of clock. (For technical or numerical purposes it might be useful to define some interpolating procedure, but this will not be done here.)

In general, it will not be possible to find points z_i such that the conditions above are satisfied exactly and we can only require that $V[z_i, z_{i+1}] \approx \gamma$ "as good as possible". This statement can be made more precise by formulating a variational principle for the choices of $\{z_i\}$. Replace (iii) above by the condition that

$$S = \sum_{i=0}^{N-1} \left(V[z_i, z_{i+1}] - \gamma\right)^2 \tag{4}$$

is minimal.

This assignment of a proper time to a directed path is approximative in at least two respects. First, if $V[x,y] < \gamma$, it does not make sense to assign a precise proper time distance to x and y. In this case we only may say that the distance (when measured with a γ-clock) is smaller than 1. Second, even if the proper time of a section of C is large, we can only determine its value up to one unit of this γ-clock. Both properties are to be expected when time "is read from a clock".

Let us now see how the given definition of proper time works for the two-dimensional light-cone lattice. Figure 2 shows four points having equal proper time distance from $(0,0)$ for $\gamma = 34$ (which is very small, cf. the numbers given below). The approximation of the hyperbolic structure of an equal-time slice is better for larger values of γ. Problems seem to occur when we approach the

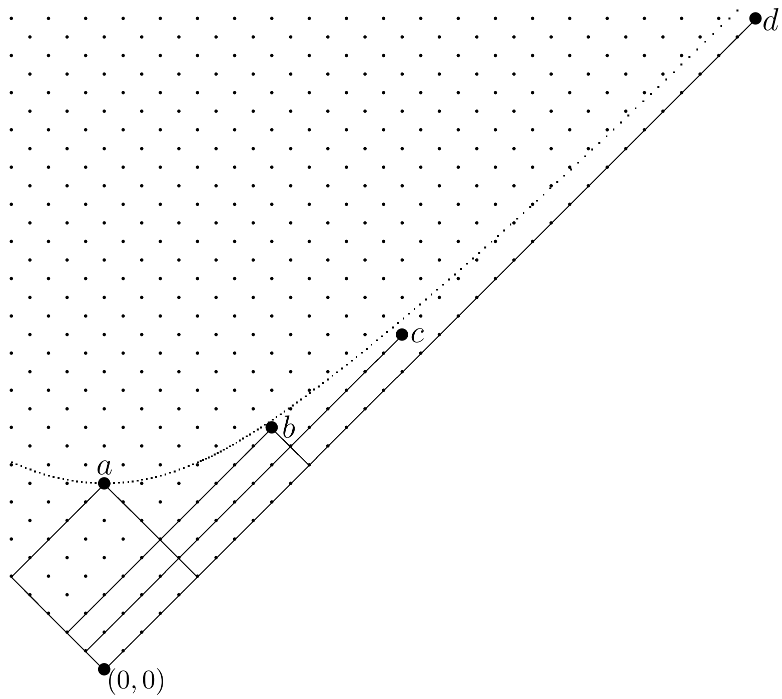

Figure 2: Alexandrov sets of equal "volume" $\gamma = 34$. The dotted curve represents the Minkowski equal time slice.

light-cone, but this is to be expected: If a clock moves relativ to an observer with a velocity such that its spacial extend by Lorentz contraction approaches the size of the lattice spacing (for instance Planck length) we expect to see deviations from continuum physics.

A more severe obstacle seems to be that for finite γ there will be an event *on* the light-cone for which the Alexandrov volume is equal to γ (event d in fig. 2). Hence, paths along the light-cone have a non-vanishing proper time. The light-cone structure seems to be a general problem for models with discretized spacetime.

But before we discard this approach completely let us put in some more realistic numbers. Assume the lattice scale to be equal to the Planck scale ($l_P \approx 10^{-33}\,\mathrm{cm}$ and $t_P \approx 10^{-44}\,\mathrm{sec}$). At present we probe physics below $1\,\mathrm{TeV} \approx 10^{-17}\,\mathrm{cm}$. (That does not imply that we can measure space and time with this accuracy.) This is 16 orders of magnitude above Planck scale. A clock (in 4 dimensional spacetime) with this precision would correspond to a value of $\gamma = 10^{64}$. The associated linear distance on the light-cone would be $10^{64} \cdot l_P \approx 10^{31}\,\mathrm{cm}$. This is several orders of magnitude larger than the present radius of the universe. So we are still far away from observing deviations from continuum physics. (However, an increase of 3 orders of magnitude in energy would correspond to a linear distance on the light-cone of about 10^{19} cm or 10 parsec, which can be measured directly by using the annual parallax of the earth. Maybe we are not so far away from seeing Planck scale physics.)

We might also have argued that this is just an unphysical artifact of our prescription and that a better understanding of the dynamics related to causal graphs will solve this problem. In any case, physical clocks cannot be accelerated to travel *on* the light cone.

Finally, we want to address the question of how to choose proper values for γ. In principle, γ may be any non-negative integer starting from 0 (in which case the above definition of proper time reduces to the mathematical proper time – the "number of links"). Therefore, γ labels an infinite family of definitions of proper time. The natural question is if these families give rise to the same geometry (up to an overall factor). In general, this will not be the case. If γ is too small, artifacts of the discreteness of spacetime may have an influence on the large scale structure, as we have seen in our discussion of the light-cone problem. For numerical purposes γ should be larger than the mathematical diameter (the length of the longest directed path) of the causal graph.

γ should not be too large either, as in this case the large scale geometry is washed out. Macroscopic curvature effects, i.e. the realm of classical (non-quantum) general relativity, should be negligible inside a world-volume of size γ. A similar requirement has to be made for all time measuring instruments in general relativity. Hence, apart from regions close to singularities (e.g. inside of black holes), this leaves a large range for γ in our world.

On the other hand, if for a causal graph we find a range of γ such that within this range a change of γ defines the same geometry (up to a scale), we may say that this range probes the continuum limit of the graph.

Up to now we have treated the world-volume of a clock during one period as an Alexandrov set. In physical applications (or for the breather solution which we will discuss in the next section), the actual world-volume might have a different shape. In any case, this volume is Lorentz invariant and in one-to-one correspondence to proper time. We have chosen Alexandrov sets just for convenience.

5 Breathers in the discretized Sine-Gordon model as an example for dynamical clocks

We now want to illustrate the concepts defined in the previous section by a simple example: the Sine-Gordon model on a 2-dim. light-cone lattice. This model has been studied in [8]. The Sine-Gordon theory in two dimensions is known to have periodic solutions - the so-called breather solutions [14] - which may serve as dynamical clocks.

The field equation of the Sine-Gordon model is:

$$\frac{\partial^2 \varphi}{\partial t^2} - \frac{\partial^2 \varphi}{\partial x^2} + g \sin\varphi \;=\; 0 \;, \tag{5}$$

where φ denotes an angular variable.

We now discretize the above equation for a light-cone lattice (for the labeling of points cf. fig. 3):

$$\begin{aligned} 0 \;=\;& \varphi(m+1,n+1) + \varphi(m,n) - \varphi(m,n+1) - \varphi(m+1,n) + \qquad (6) \\ & \frac{1}{4} g \left[\sin\varphi(m,n) + \sin\varphi(m{+}1,n) + \sin\varphi(m,n{+}1) + \sin\varphi(m{+}1,n{+}1)\right] \;. \end{aligned}$$

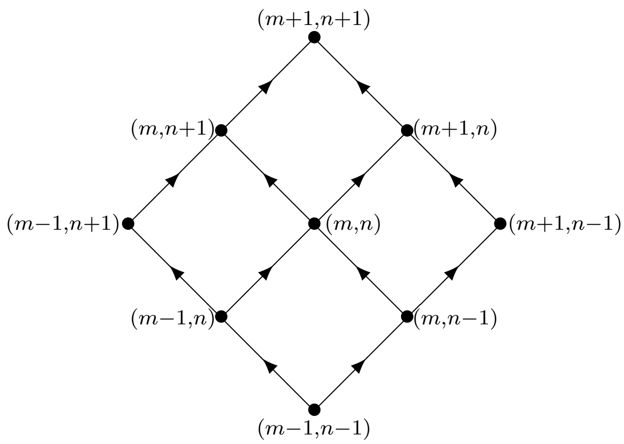

Figure 3: Labeling of the vertices on a 2-dimensional light-cone lattice

We are looking for solutions for which the values of the fields on neighbored vertices do not differ much. Such weakly fluctuating solutions are obtained for $g \ll 1$. In this case the solutions will be given in good approximation by the continuum equations.

We have labeled the points of the light-cone lattice by $(m,n) \simeq (t-x, t+x)$ which refers to a fixed background spacetime with a Newtonian character. We know, however, that the continuum equations are invariant under Lorentz transformations. This invariance is not an invariance of the spacetime structure defined by the lattice, but a property of the set of solutions of the equation: If $\varphi(x,t)$ is a solution of eq. (5), then

$$\hat{\varphi}(x,t) \;=\; \varphi\left(\,\gamma(v)(x-vt)\;,\;\gamma(v)(t-vx)\,\right) \quad \text{with} \;\; \gamma(v) = \frac{1}{\sqrt{1-v^2}} \tag{7}$$

is also a solution of this equation for arbitrary $-1 < v < 1$. As long as $g\gamma(v) \ll 1$, the solutions of the discretized equation (6) will be approximated by the solutions of the continuum equation. Within the framework of this approximation the set of discretized solutions will share the same invariance property (7). Hence, *not the structure or labeling of the lattice but the invariance of the space of solutions of the equations of motion - the dynamics - will define "intrinsic spacetime".*

The breather solutions of the Sine-Gordon equation may be interpreted as metastable bound states of a soliton and an anti-soliton. At rest this solution is given by

$$\varphi(x,t) \;=\; -4\tan^{-1}\left[\frac{a}{\sqrt{1-a^2}}\frac{\sin\sqrt{g(1-a^2)}(t-t_0)}{\cosh a\sqrt{g}(x-x_0)}\right] . \tag{8}$$

The parameter a has to satisfy the condition $0 < a^2 < 1$ but is arbitrary otherwise. The period for this solution is

$$\Delta T_0 \;=\; \frac{2\pi}{\sqrt{g(1-a^2)}} . \tag{9}$$

In order to employ the breather solution as a dynamical clock we have to choose a definite value for a, thereby setting a time scale. A possible choice corresponds to the case where φ varies between $-\pi$ and $+\pi$, i.e. $a^2 = 1/2$ or $\Delta T_0 = \sqrt{8\pi^2/g}$.

We may use the invariance of the field equation to obtain a breather solution which moves with a velocity v. Not surprisingly, the period as seen by an external observer (like us) changes to

$$\Delta T_v \;=\; \frac{1}{\sqrt{1-v^2}}\Delta T_0 . \tag{10}$$

The period of a moving bound state is longer by a factor of $\gamma(v)$ with respect to the period of a breather solution at rest. On the other hand, the width of the breather solution is Lorentz contracted such that the world-volume during one period (defined, e.g., by the set of events for which $|\varphi|$ is larger than some constant) is unchanged. This world volume does not resemble an Alexandrov set at all (cf. fig. 4), but it is in one-to-one correspondence to proper time. Measuring time means counting elementary volumes.

Intrinsically, the breather solution defines the proper time distance between two timelike events. This structure is obviously the Minkowski structure (as long as we do not approach the light-cone too closely).

6 Concluding remarks

We presented a definition of proper time for causal graphs which is based on the idea of dynamical clocks with a certain accuracy. We have seen that

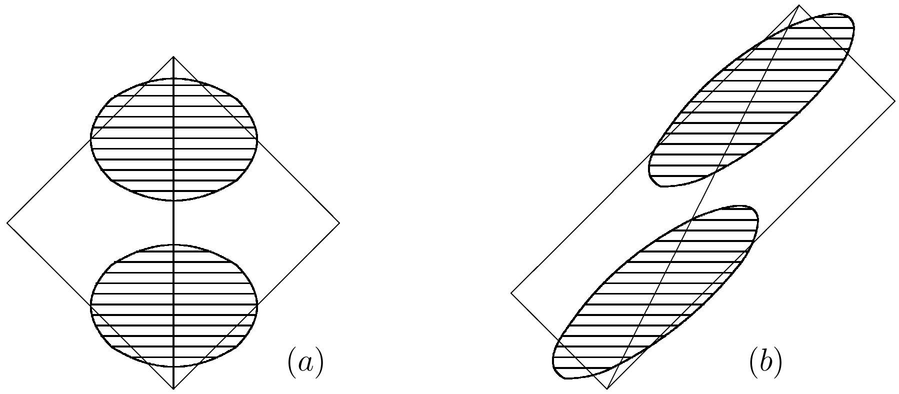

Figure 4: World-volumes of the breather solution for (a) $v = 0$ and (b) $v = 0.5$ and the corresponding Alexandrov sets. (The shaded areas represent events for which $|\varphi| > 1.5$.)

- the assignment of a proper time to a section of a directed path depends on the world volume γ of a clock during one "tick" and is approximative because the resolution of proper time has a natural cut-off depending on the type of clock.

- the geometry of the causal graph may depend on this resolution. If the accuracy of the clock can resolve the discrete nature of the causal graph, the large scale structure deviates from that of a Minkowski space. In the other extreme the clock may smear out large scale curvature dependencies. Only if the one-tick world volume of the clock lies within the range where a Lorentz approximation to spacetime is valid, the reconstructed geometry corresponds to the one of general relativity.

- the breather solution of the Sine-Gordon equation is an example of a clock moving on the light-cone lattice (which is a causal graph). However, as emphasized before, the full theory should generate spacetime and propagating fields simultaneously.

A few remarks are in order:

- Although we have used the language of causal graphs, the concept may be applied to any discrete structure with a causality relation. It might be especially well suited for causal sets.

- One might wonder why the causal structure is not sufficient for a Minkowski or Lorentz structure to arise at large scales, which is what many theorems on causal spaces imply [15, 16, 17]. (The notion of "volume" - number of events - should fix the metric even completely.) However, in most cases at least continuity or the concept of parallel lines is required to prove these theorems.

- Apart from the physically motivated definition of proper time, this mechanism might also be an alternative way to define coarse graining for causal graphs and causal sets, which is important for the definition of a continuum limit. In our approach, the assignment of proper times implies the continuum limit, as larger values of γ probe larger scales of the causal graph.

  It would be interesting to see, if there is a relation (or even equivalence) to the coarse graining procedure proposed by the authors of [13]. They construct a coarse grained causal set from a random selection of events, which resembles the "decimation" in the context of spin systems. The approach presented here rather resembles a block spin transformation where an effective magnetization is assigned to a certain volume which then represents a point in the coarse grained lattice. However, we never select specific volumes to obtain a coarse grained lattice (which might violate Lorentz invariance) but rather leave the causal graph unchanged and only probe it on different scales. This procedure is closer to what we actually do in Nature.

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